Relativistic Hydrodynamics for Heavy–Ion Collisions II. Compression of Nuclear Matter and the Phase Transition to the Quark–Gluon Plasma[†]

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Abstract

We investigate the compression of nuclear matter in relativistic hydrodynamics. Nuclear matter is described by a $\sigma - \omega$–type model for the hadron matter phase and by the MIT bag model for the quark–gluon plasma, with a first order phase transition between both phases. In the presence of phase transitions, hydrodynamical solutions change qualitatively, for instance, one-dimensional stationary compression is no longer accomplished by a single shock but via a sequence of shock and compressional simple waves. We construct the analytical solution to the "slab-on-slab" collision problem over a range of incident velocities. The performance of numerical algorithms to solve relativistic hydrodynamics is then investigated for this particular test case. Consequences for the early compressional stage in heavy–ion collisions are pointed out.

1 Introduction

To investigate heavy–ion collision dynamics in realistic, i.e., 3+1–dimensional, situations by means of hydrodynamics requires numerical schemes to solve the hydrodynamical equations of motion. These schemes should reproduce analytical solutions, as far as such exist at all. In a previous paper [1] we have presented two algorithms for ideal relativistic hydrodynamics, the SHASTA, a flux–corrected transport algorithm [2], and the relativistic HLLE, a Godunov–type algorithm [3]. We have investigated their performance for the expansion of matter into the vacuum, where they are confronted with two problems generical for simulations of heavy–ion collisions. These are the presence of vacuum itself and the qualitative change in the type of the hydrodynamical expansion solution if phase transitions occur in the equation of state (EoS).

As was explained in detail in [1, 4], matter that undergoes a first order phase transition may exhibit thermodynamically anomalous behaviour in a certain range of independent thermodynamic variables, signalled by a change of sign of the quantity

$$\Sigma \equiv \left.\frac{\partial^2 p}{\partial \epsilon^2}\right|_\sigma + 2\, c_s^2\, \frac{1 - c_s^2}{\epsilon + p}\ , \tag{1}$$

where $c_s^2 \equiv \partial p/\partial \epsilon|_\sigma$ is the velocity of sound, ϵ, p, and σ are the energy density, pressure, and specific entropy, respectively. For thermodynamically normal (TN) matter, $\Sigma > 0$, for so-called thermodynamically anomalous (TA) matter $\Sigma < 0$ [1, 4].

As was shown in [1, 4, 5], for the one–dimensional expansion of TN matter a simple rarefaction wave is the stable hydrodynamical solution, while for TA matter such a wave is unstable with respect to formation of a rarefaction shock wave. Moreover, an initial discontinuity is bound to decay into a simple wave in TN matter, but cannot do so if matter is TA (or even if Σ only vanishes instead of becoming negative). Thus, rarefaction discontinuities form the stable hydrodynamical solution in the expansion of TA matter.

On the other hand, for the compression of TA matter a compressional simple wave forms the stable hydrodynamic solution, but it is unstable in TN matter with respect to formation of a compressional shock wave. Analogously, a compressional shock discontinuity is bound to decay into a compressional simple wave in TA matter, while it cannot do so in TN matter. Thus, compression shocks are stable in the compression of TN matter.

As outlined in [1], in realistic cases the EoS has both TN and TA regions. Consequently, the hydrodynamical solution for the one–dimensional expansion is more complicated, involving a sequence of simple waves, regions of constant flow, and discontinuities. It was shown in [4] that the same holds for the compression. The corresponding hydrodynamical solution was explicitly constructed in the case of a one–dimensional "slab-on-slab" collision. (For TN matter this test problem and the corresponding performance of the SHASTA and relativistic HLLE algorithms was studied in [3].) The nuclear matter equations of state used in [4] featured a first order phase transition between the quark–gluon plasma (QGP), described by the MIT bag model, and hadronic matter, described by phenomenological equations of state [6, 7].

In this work we first construct a nuclear matter EoS similar to that of Ref. [4]. We also use the MIT bag EoS [8] for the QGP phase, but for the hadronic phase, we employ a version of the $\sigma - \omega$–model [9] (plus massive thermal pions) which features more realistic values for the ground state incompressibility and the effective nucleon mass than the original version proposed by Walecka [6]. For this EoS, we construct analytically the hydrodynamical compression solution as described in [4]. With a tabulated version of the EoS, we then test the ability of the numerical algorithms to reproduce this solution. This does not only yield insight into the usefulness of these algorithms for applications in heavy–ion collisions and dynamical studies of QGP formation. In a sense it also represents an independent "numerical" proof for the theoretical arguments of Ref. [4] about the form of the hydrodynamical solution for compression of TA matter.

This paper is organized as follows. In Section 2 we present our nuclear matter EoS. In Section 3 we analytically construct the hydrodynamic solution for a one–dimensional "collision" of semi-infinite slabs. Section 4 comprises our numerical results. In Section 5 we comment on one–dimensional collisions of finite nuclei and draw conclusions for numerical simulations of heavy–ion collisions. Section 6 concludes this work with a summary of our results. An Appendix contains results for modifications of the standard algorithms used throughout Ref. [1] and this work. We use natural units $\hbar = c = k_B = 1$.

2 The nuclear matter EoS

2.1 Hadron matter

For hadronic matter we use a modification of the original $\sigma - \omega$–model [6], which was presented in Ref. [9]. For the original $\sigma - \omega$–model, the EoS, i.e., the pressure p as a function of the independent thermodynamical variables temperature T and baryochemical potential μ, can be rigorously derived from the $\sigma - \omega$–Lagrangian employing the mean–field (or Hartree, or one–loop) approximation of quantum many–body theory at finite temperature and density [10], see for instance [11] for an explicit derivation. A more general class of thermodynamically consistent, phenomenological equations of state for (non–strange) interacting nucleonic matter is defined by [12]

$$p_{had}(T, \mu) = p_N(T, \nu; M^*) + p_N(T, -\nu; M^*) + \sum_i p_i(T; m_i) \tag{2}$$

$$+ n\, \mathcal{V}(n) - \int_0^n \mathcal{V}(n')\, \mathrm{d}n' - \rho_s\, \mathcal{S}(\rho_s) + \int_0^{\rho_s} \mathcal{S}(\rho_s')\, \mathrm{d}\rho_s' \, .$$

Here,

$$p_N(T, \nu; M^*) = T\, \frac{g_N}{(2\pi)^3} \int \mathrm{d}^3\mathbf{k}\ \ln\left[1 + \exp\left\{-\left(E_k^* - \nu\right)/T\right\}\right] \tag{3}$$

is the pressure of an ideal gas of nucleons (spin–isospin degeneracy $g_N = 4$) moving in the scalar potential $\mathcal{S}$ and the vector potential $\mathcal{V}$ [1]. These potentials generate an *effective*

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[1] More accurately, $\mathcal{V}$ is the zeroth component of the vector potential $\mathcal{V}^\mu$, the spatial components of which vanish in a homogeneous, isotropic system.

nucleon mass

$$M \quad \longrightarrow \quad M^* \equiv M - \mathcal{S}(\rho_s) \ , \tag{4}$$

where $M = 938$ MeV is the nucleon mass in the vacuum, and also shift the one–particle energy levels

$$E_k \equiv \sqrt{\mathbf{k}^2 + M^2} \quad \longrightarrow \quad E_k^* + \mathcal{V}(n) \equiv \sqrt{\mathbf{k}^2 + (M^*)^2} + \mathcal{V}(n) \ . \tag{5}$$

The vector potential is conveniently absorbed in the *effective* chemical potential

$$\nu \equiv \mu - \mathcal{V}(n) \ , \tag{6}$$

giving rise to the interpretation of (3) as the pressure of an ideal gas of *quasi-particles* with mass M^* and chemical potential ν. Furthermore,

$$p_i(T; m_i) = -T \frac{g_i}{(2\pi)^3} \int \mathrm{d}^3\mathbf{q} \ \ln\left[1 - \exp\left\{-\sqrt{\mathbf{q}^2 + m_i^2}/T\right\}\right] \tag{7}$$

is the pressure of an ideal gas of mesons with degeneracy g_i and mass m_i. In the following, we will only include pions ($g_\pi = 3$, $m_\pi = 138$ MeV), since they are the lightest and thus most abundant mesons. (To be consistent, one should also include thermal contributions for the σ and ω meson [11]. However, since these mesons are much heavier than the pion, one can neglect their contribution for the present applications.) n is the (net) baryon density,

$$
\begin{aligned}
n(T, \mu) &\equiv \left.\frac{\partial p}{\partial \mu}\right|_T \\
&= \frac{g_N}{(2\pi)^3} \int \mathrm{d}^3\mathbf{k} \ \left[\frac{1}{e^{(E_k^* - \nu)/T} + 1} - \frac{1}{e^{(E_k^* + \nu)/T} + 1}\right] \ ,
\end{aligned}
\tag{8}
$$

and

$$\rho_s(T, \mu) \equiv \frac{g_N}{(2\pi)^3} \int \mathrm{d}^3\mathbf{k} \ \frac{M^*}{E_k^*} \ \left[\frac{1}{e^{(E_k^* - \nu)/T} + 1} + \frac{1}{e^{(E_k^* + \nu)/T} + 1}\right] \tag{9}$$

is the scalar density of nucleons. It is assumed that the vector potential (which transforms like the zeroth component of a vector) depends *only* on the (net) baryon density (which also transforms like the zeroth component of a vector, namely the (net) baryon current N^μ), while the scalar potential (which is a Lorentz–scalar) depends only on the (Lorentz–) scalar density ρ_s.

Once the pressure is known, the entropy and energy density can be obtained from the thermodynamical relations,

$$s = \left.\frac{\partial p}{\partial T}\right|_\mu \ , \quad \epsilon = Ts + \mu n - p \ . \tag{10}$$

One now has to specify the potentials $\mathcal{V}, \mathcal{S}$. The choice

$$\mathcal{V}(n) = C_V^2 \, n \ , \quad \mathcal{S}(\rho_s) = C_S^2 \, \rho_s \tag{11}$$

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reproduces the original $\sigma - \omega$–model [11]. When fitting the two parameters C_V^2, C_S^2 to reproduce the experimental values for the ground state binding energy of infinite nuclear matter, $B_0 = 16$ MeV, and the ground state density, $n_0 = 0.15891\,\text{fm}^{-3}$ [13], the effective mass in the nuclear ground state is $M_0^* \simeq 0.543\,M$ which is too small (the experimental value ranges around $0.7\,M$ [14]), and the incompressibility is $K_0 \equiv 9\,\text{d}p/\text{d}n|_{n_0} \simeq 553$ MeV which is too large by about a factor of two [15]. To adjust this shortcoming of the model, it was suggested to introduce self-interaction terms, $\sim \sigma^3$, σ^4, for the scalar σ meson field in the $\sigma - \omega$–Lagrangian [16]. This nonlinear $\sigma - \omega$–model has two additional parameters which enable one to also independently adjust M_0^* and K_0 to the experimentally observed values.

Alternatively, in Ref. [9] it was suggested to use

$$\mathcal{V}(n) = C_V^2\, n - C_d^2\, n^{1/3} \ , \quad \mathcal{S}(\rho_s) = C_S^2\, \rho_s \ . \tag{12}$$

Although this choice introduces only *one* additional parameter, fitting the ground state incompressibility gives *simultaneously* reasonable values for the effective mass, for details see [9]. In the following we use the parameters $C_V^2 = 238.08\,\text{GeV}^{-2}$, $C_S^2 = 296.05\,\text{GeV}^{-2}$, $C_d^2 = 0.183$, which leads to $M_0^* = 0.635\,M$, $K_0 = 300$ MeV.

2.2 Quark–Gluon Plasma

For the QGP EoS we employ the standard MIT bag model [8] for massless, non-interacting gluons and u, d quarks, i.e.,

$$p_{QGP}(T, \mu) = \frac{37\pi^2}{90}\, T^4 + \frac{1}{9}\, \mu^2 T^2 + \frac{1}{162\pi^2}\, \mu^4 - B \ , \tag{13}$$

and other thermodynamical quantities follow again from the relations (8, 10). Note that p does not depend explicitly on n for this EoS,

$$p = \frac{1}{3}(\epsilon - 4B) \ . \tag{14}$$

For the bag constant, we use the value $B = (235\,\text{MeV})^4$ throughout this work. This value results in a phase transition temperature of $T_c \simeq 169$ MeV at vanishing baryon density (see below), which is roughly in agreement with what lattice QCD simulations predict [17].

2.3 Gibbs Phase Equilibrium and Thermodynamical Phase Diagrams

The QGP EoS (13) is matched to the hadronic EoS (2) with (12) via Gibbs' conditions for (mechanical, thermal, and chemical) phase equilibrium,

$$p_{had} = p_{QGP} \ , \quad T_{had} = T_{QGP} \ , \quad \mu_{had} = \mu_{QGP} \ , \tag{15}$$

which leads to a phase boundary curve $T^*(\mu^*)$ in the $T - \mu$ plane defined by the implicit equation $p_{had}(T^*, \mu^*) = p_{QGP}(T^*, \mu^*)$, see Fig. 1 (a). Along this curve, one can calculate the phase boundary values for other thermodynamical variables as a function of T^*. Fig. 1 (b) shows the phase boundaries $n_{H,Q}(T^*)$ in the $T - n/n_0$ phase diagram, Fig. 1 (c) the phase boundaries $\epsilon_{H,Q}(T^*)$ in the $\epsilon/\epsilon_0 - T$ diagram, and Fig. 1 (d) $\epsilon_H(n_H(T^*))$ and $\epsilon_Q(n_Q(T^*))$ in the $\epsilon/\epsilon_0 - n/n_0$ diagram, respectively ($\epsilon_0 = (M - B_0)n_0$ is the ground state energy density). The phase transition constructed via (15) is of first order, leading to a mixed phase of QGP and hadron matter and to a latent heat, as can be seen in Figs. 1 (b,c,d). The T–axis of Fig. 1 (b) maps onto the (dotted) curve $\epsilon(T, n = 0)$ in Fig. 1 (c). Correspondingly, the ϵ/ϵ_0–axis in Fig. 1 (c) maps onto the (dotted) curve $\epsilon(n, T = 0)$ in Fig. 1 (d). This zero–temperature energy density is finite for finite n due to the Fermi (and, in the hadronic phase, interaction) energy of the fermions in the system (nucleons and quarks, respectively). This curve represents the *minimum* energy density possible for a given baryon density.

As described in [1], the pressure as a function of energy density and baryon density, $p(\epsilon, n)$, is required for solving the hydrodynamical equations numerically. It is convenient to tabulate this function, since calculating the pressure in the hadron phase for given ϵ, n requires a triple root search (a double root search to find T, μ for given ϵ, n, while simultaneously solving a fixed-point equation for the effective nucleon mass). The computational effort would be prohibitive if one tries to perform this "on-line" for each hydrodynamic cell in each time step of the hydrodynamical transport algorithm.

Therefore, we discretize the $\epsilon - n$ plane of Fig. 1 (d) in the range $0 \leq \epsilon/\epsilon_0 \leq 20$, $0 \leq n/n_0 < 12$ with equidistant grid spacing $\Delta\epsilon = \epsilon_0/10$, $\Delta n = n_0/20$ to form a 201×240 mesh. This grid spacing proves to be sufficiently small to use the resulting table also for analytical calculations, see Section 4. The phase boundaries $\epsilon_{H,Q}$, as well as the $\epsilon(n, T = 0)$–curve are mapped onto the 240 grid points of the n/n_0–axis ($\epsilon_{H,Q}$ is taken to be zero for n–values exceeding those where the phase boundaries meet the $\epsilon(n, T = 0)$–curve).

Now values for the pressure are assigned to the mesh points. In the hadronic phase (defined by $\epsilon(n, T = 0) \leq \epsilon \leq \epsilon_H(n)$ for given n) this is done using eq. (2) and the mentioned triple root search, and in the QGP phase (defined by $\max\{\epsilon_Q(n), \epsilon(n, T = 0)\} \leq \epsilon \leq 20\,\epsilon_0$ for given n) eq. (14) is employed. In the mixed phase ($\max\{\epsilon_H(n), \epsilon(n, T = 0)\} \leq \epsilon \leq \epsilon_Q(n)$ for given n) the pressure is calculated as follows. For given T^*, the values of energy and baryon density read

$$\epsilon = \lambda_Q \epsilon_Q(T^*) + (1 - \lambda_Q)\epsilon_H(T^*) \, , \tag{16}$$

$$n = \lambda_Q n_Q(T^*) + (1 - \lambda_Q)n_H(T^*) \, , \tag{17}$$

where λ_Q is the fraction of volume the QGP phase occupies in the mixed phase. Vice versa, for given ϵ, n these equations yield values for λ_Q, T^*. Eliminating λ_Q one obtains a single equation for T^* as a function of ϵ, n. This is solved numerically using the values $\epsilon_{H,Q}(T^*)$, $n_{H,Q}(T^*)$ of Figs. 1 (b,c). Once T^* is known, μ^* follows from Fig. 1 (a), which consequently yields the pressure from Gibbs' phase equilibrium conditions (15).

Similarly, every other thermodynamic quantity can be calculated as a function of ϵ and n. Of particular importance are temperature, baryo-chemical potential, and entropy density. Once the first two are known, the entropy density is obtained from the second equation (10).

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In the hadronic phase, T and μ emerge naturally from the triple root search, while in the mixed phase T^* is obtained as described above and μ^* then follows from Fig. 1 (a). In the QGP, we distinguish two cases. For $n = 0$ we also have $\mu = 0$, and then we obtain from (13, 14) the simple formula $T = [30(\epsilon - B)/37\pi^2]^{1/4}$. For finite μ we eliminate T from the equation of ϵ using the equation for n. This results in a sixth order equation in μ,

$$0 = \frac{4}{1215\pi^2}\,\mu^6 - \frac{4}{15}\,n\mu^3 + (\epsilon - B)\mu^2 - \frac{999\pi^2}{40}\,n^2 \;, \tag{18}$$

which has to be solved numerically. After that, $T = [9n/2\mu - \mu^2/9\pi^2]^{1/2}$ (which follows from the equation for n).

In Fig. 2 we show (a) pressure, (b) temperature, (c) baryo-chemical potential, and (d) entropy density as functions of ϵ and n, as they emerge from the above described procedure. Note in Fig. 2 (a) that on account of (14) the pressure is independent of n in the QGP phase. Also, in the mixed phase for small n the pressure is (almost) constant as a function of either ϵ or n. The thermodynamical identity

$$c_s^2 \equiv \left.\frac{\partial p}{\partial \epsilon}\right|_\sigma = \left.\frac{\partial p}{\partial \epsilon}\right|_n + \frac{n}{\epsilon + p}\left.\frac{\partial p}{\partial n}\right|_\epsilon \tag{19}$$

then implies that the velocity of sound is very small in this region. For $n = 0$, the pressure (as well as the temperature, see Fig. 2 (b)) is constant in the mixed phase, and the velocity of sound vanishes identically (cf. the EoS for TA matter in [1]).

In contrast to the diagrams for the thermodynamical variables p, T, μ which enter Gibbs' phase equilibrium conditions (15), there is no obvious sign for the mixed phase (such as edges due to the phase boundaries) in the entropy density. The reason is that this quantity is a linear interpolation between s_H and s_Q (similar to (16, 17)) in this phase.

For the hydrodynamical simulations intermediate values of thermodynamic quantities are calculated by two–dimensional linear interpolation. If $\epsilon > \max\{20\,\epsilon_0, \epsilon(n, T = 0)\}$ for given n, the QGP EoS (14) is used directly to calculate the pressure, and temperature, chemical potential, and entropy density are obtained as described above.

3   Compression of nuclear matter

In this section we construct the analytical solution to the one–dimensional compression of nuclear matter with the EoS presented in Section 2. The initial condition of the "slab-on-slab" collision problem is

$$
\begin{aligned}
\epsilon(x, 0) &= \epsilon_0 \;, & -\infty < x < \infty \;, \tag{20}\\
n(x, 0) &= n_0 \;, & -\infty < x < \infty \;, \tag{21}\\
v(x, 0) &= \left\{ \begin{array}{ll} v_{CM} \;, & -\infty < x \leq 0 \;, \\ -v_{CM} \;, & 0 < x < \infty \;. \end{array} \right. \tag{22}
\end{aligned}
$$

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In a real heavy–ion collision (see also Section 5), v_{CM} corresponds to the velocity of the nuclei in the "equal–velocity frame". For identical nuclei, this is equivalent to the center-of-momentum (CM) frame. In the subsequent time evolution, momentum conservation will force matter to stop at $x = 0$ and energy conservation to convert its kinetic energy into internal energy, leading to compression waves travelling symmetrically away from the origin, leaving in their wake compressed and heated matter at rest. For small v_{CM}, the energy and baryon number densities obtained in the compressed state correspond to hadronic, i.e., TN matter. Thus, compression proceeds via single shock waves. Energy, momentum, and (net) baryon number conservation across the (space–like) shock discontinuity read in the rest frame of the shock (quantities with subscript "0" correspond to the uncompressed state)

$$T^{01} = T_0^{01} \ , \quad T^{11} = T_0^{11} \ , \quad N^1 = N_0^1 \ , \tag{23}$$

where $T^{\mu\nu} = (\epsilon + p)u^\mu u^\nu - pg^{\mu\nu}$ is the energy momentum tensor for an ideal fluid, and $N^\mu = nu^\mu$ is the (net) baryon number current. The final compressed state (ϵ, n) obeys the *Taub equation* [18]

$$wX - w_0 X_0 - (p - p_0)(X + X_0) = 0 \ , \tag{24}$$

where $w = \epsilon + p$ is the enthalpy density and $X = w/n^2$ the generalized volume[2]. The Taub equation defines the so-called *Taub adiabat* in the $p - X$ plane. It is the set of all possible final states which are in accordance with energy, momentum, and baryon number conservation across a single shock discontinuity. A particular final state is fixed by specifying the involved velocities. For instance, in our case the velocity v_{CM} of uncompressed matter in the rest frame of compressed matter is given. On the other hand, in the rest frame of the shock, uncompressed matter flows into the shock with velocity v_0 and compressed matter emerges with velocity v. These velocities obey [18]

$$v_0^2 = \frac{(p - p_0)(\epsilon + p_0)}{(\epsilon - \epsilon_0)(\epsilon_0 + p)} \ , \quad v^2 = \frac{(p - p_0)(\epsilon_0 + p)}{(\epsilon - \epsilon_0)(\epsilon + p_0)} \ . \tag{25}$$

Obviously, $v_{CM} = |v_0 - v|/(1 - v_0 v)$ and thus

$$\gamma_{CM}^2 = \frac{(\epsilon_0 + p)(\epsilon + p_0)}{w \, w_0} = \left[\frac{(\epsilon + p_0) \, n_0}{w_0 \, n} \right]^2 \equiv \left(\frac{\epsilon/n}{\epsilon_0/n_0} \right)^2 \ , \tag{26}$$

since $p_0 \equiv 0$ in the ground state of nuclear matter. In deriving this equation we have made use of the Taub equation (24). Given v_{CM}, this equation fixes ϵ/n, or with the Taub equation, ϵ *and* n in the final compressed state.

Fig. 3 (a) shows the Taub adiabat with the center $(X_0, p_0) = (\epsilon_0/n_0^2, 0)$ for the EoS of Section 2. The different parts belonging to final states in the hadron, mixed, and QGP phase are marked correspondingly. One observes a region of final states (A–C) where the chord connecting them with the center intersects the Taub adiabat. For these final states a single compression shock is not the hydrodynamically stable solution [4, 12, 19].

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[2]Note that this definition differs from the one given in [1] for the (net) baryon-free case.

To see this and to determine the stable solution one first notes that point A is a Chapman-Jouguet (CJ) point of the Taub adiabat. At this point, the specific entropy $\sigma \equiv s/n$ assumes a (local) maximum and the slope of the Taub adiabat and the Poisson adiabat (the curve of constant specific entropy) are identical [19]

$$\left.\frac{\partial X}{\partial p}\right|_{\sigma} \equiv \left.\frac{\partial X}{\partial p}\right|_{TA} . \tag{27}$$

Furthermore, above the CJ point the chord connecting a final state (X, p) with the center has a smaller slope than the Taub adiabat at (X, p), i.e.,

$$\left.\frac{\partial X}{\partial p}\right|_{TA} \leq \frac{X - X_0}{p - p_0} \;\;,\;\; p > p_{CJ} , \tag{28}$$

with the equality holding at the CJ point. Subtracting this equality from (28), we obtain

$$\frac{\partial X/\partial p|_{TA,\,p} - \partial X/\partial p|_{\sigma,\,p_{CJ}}}{p - p_{CJ}} \leq 0 \;\;,\;\; p > p_{CJ} . \tag{29}$$

In the limit $p \to p_{CJ}$ we conclude using (27):

$$\left.\frac{\partial^2 X}{\partial p^2}\right|_{\sigma,\,p_{CJ}} \equiv \tilde{\Sigma} \leq 0 \;\; \text{for} \;\; p > p_{CJ} . \tag{30}$$

From thermodynamical identities, $\tilde{\Sigma} \equiv \Sigma/n^2 c_s^6$, and therefore, *above the CJ point matter is TA*. This implies that, once the CJ point is reached via a single compression shock, further compression can be achieved only through a *compressional simple wave*. The hydrodynamical solution will therefore consist of a single compression shock to the CJ point and, attached to it, a compressional simple wave (see Fig. 5 below). Since matter emerges from the shock with the velocity of sound (since the compressed state corresponds to the CJ point [19]), and since for a simple wave the matter velocity relative to the wave profile is also the velocity of sound, the simple wave profile does not move relative to the shock, and they will remain attached to each other in the stationary state. Since the specific entropy σ is constant for continuous solutions of ideal hydrodynamics, final states of compressed matter lie on the Poisson adiabat through the CJ point, rather than on the Taub adiabat, see Fig. 3 (b).

Compression of matter can proceed through this configuration only until the point of inflection, $\partial^2 X/\partial p^2|_\sigma = 0$, on the Poisson adiabat is reached (point B in Fig. 3 (b); this is usually the point where the Poisson adiabat leaves the mixed phase and enters the QGP phase). Since matter becomes TN at this point, further compression is achieved by another compression shock attached to the head of the compressional simple wave (see Fig. 6 below). This shock is also described by the Taub equation (24), however, X_0, p_0, and w_0 have to replaced by the corresponding quantities $\tilde{X}$, $\tilde{p}$, and $\tilde{w}$ at the head of the compressional simple wave. The final compressed state lies in the pure QGP phase and is determined as follows. For a stationary configuration, the (baryon) flux $N^1 = n\gamma v$ at the head of the compressional simple wave has to be identical to that through the shock front. In the rest frame of the

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latter it is determined by $(N^1)^2 = -(p - \tilde{p})/(X - \tilde{X})$ (which follows from the conservation laws (23); the interpretation is that the square of the baryon flux through the shock equals the negative slope of the chord connecting initial and final state of the shock). On the other hand, the matter velocity at a fixed point of a simple wave profile is equal to the velocity of sound, i.e., in the rest frame of that point the baryon flux is $N^1 = \tilde{n}c_s/(1 - c_s^2)^{1/2}$. Now consider this point to be the head of the simple wave, or equivalently, the position of the compression shock, if the configuration is stationary. Since $\partial p/\partial X|_\sigma = -n^2 c_s^2/(1 - c_s^2)$ (which follows from thermodynamical identities), we arrive at the condition

$$\left. \frac{\partial p}{\partial X} \right|_{\sigma, \tilde{X}} \equiv \frac{p - \tilde{p}}{X - \tilde{X}} \tag{31}$$

for a stationary hydrodynamical configuration. This means that the final compressed state (X, p) on the Taub adiabat with center $(\tilde{X}, \tilde{p})$ is determined by the intersection of this adiabat with the tangent to the Poisson adiabat at $(\tilde{X}, \tilde{p})$.

In order to reach higher compression through this shock, one has to decrease the slope (31), i.e., the state $(\tilde{X}, \tilde{p})$ has to "move backwards" along the Poisson adiabat towards the CJ point A. The compressional wave will consequently become "shorter" and the compression shock stronger. This proceeds until the CJ point A is reached. Then, the compressional simple wave vanishes and the shock (O–A) merges with the shock (A–C). Since the chord (A–C) has the same slope as the chord (O–A), the baryon flux through the two shocks is identical and therefore, the single shock (O–C) becomes the hydrodynamically stable, stationary configuration. Above C, the single shock described by the Taub adiabat of Fig. 3 (a) represents again the stable hydrodynamical solution.

In practice, one obtains part (B–C), the so-called *wave adiabat* [4], when calculating the Poisson adiabat: for each $(\tilde{X}, \tilde{p})$ one simply solves (31) simultaneously with the Taub equation with center $(\tilde{X}, \tilde{p})$. The set of all final compressed states (X, p) thus obtained was called *generalized shock adiabat* in Ref. [4].

Let us now complete the solution of the hydrodynamical problem. As was the case for the expansion of semi–infinite matter studied in [1], the stationary hydrodynamical solution is of similarity type, i.e., its profile is constant as a function of $\zeta \equiv x/t$. Due to causality and the symmetry of the problem, it suffices to consider the range $0 \leq \zeta \leq 1$. Below and at the CJ point, compression proceeds through a single shock, travelling to the right with velocity $v_{sh} \equiv -v$, where v is given by (25). Given v_{CM}, the final state energy density ϵ and baryon number density n are obtained as solutions of (24) and (26). Other hydrodynamical variables can be inferred from the EoS and the definition of $T^{\mu\nu}$ and N^μ. Profiles of T^{00}/ϵ_0 and T as functions of ζ are shown in Fig. 4 for $v_{CM} = 0.7$.

The CJ point itself has to be determined numerically by the requirement of maximum specific entropy. For the above EoS, $p_{CJ} \simeq 1.941\,\epsilon_0$, $X_{CJ} \simeq 0.422\,X_0$, ($\epsilon_{CJ} \simeq 6.971\,\epsilon_0$, $n_{CJ} \simeq 4.596\,n_0$,) with a specific entropy $\sigma \simeq 3.545$. For further purpose, let us denote the value of v_{CM} required to reach the CJ point by v_{CJ}. From (26) we infer $v_{CJ} \simeq 0.752$.

Above the CJ point and below point B, compression proceeds through a shock to the CJ point and an attached compressional simple wave. The shock moves with $v_{sh} = [v'_{sh} - v_{CM}]/[1 - v'_{sh}\,v_{CM}]$ where v'_{sh} is the shock velocity in the rest frame of incoming matter. The

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latter is identical with $-v_0$, cf. (25), i.e., $v'_{sh} \simeq 0.878$. Obviously, matter emerges from the shock with velocity v_{CJ} in the rest frame of incoming matter, i.e., in the CM frame matter emerges from the shock with velocity $[v_{CJ} - v_{CM}]/[1 - v_{CJ}v_{CM}]$. This is, of course, still negative above the CJ point, since the subsequent compressional simple wave will further decelerate matter.

Consider the compressional simple wave moving to the left (i.e., that at $x > 0$) in the rest frame of matter emerging from the shock. Constancy of the Riemann invariant $\mathcal{R}_-$ for this wave [1] implies that the velocity as a function of the energy density reads

$$v''_w(\epsilon_w) = \tanh\left\{ \int_{\epsilon_{CJ}}^{\epsilon_w} d\epsilon' \; \frac{c_s(\epsilon')}{\epsilon' + p(\epsilon')} \right\} \; . \tag{32}$$

The integral is calculated via a centered Riemann sum. The required values of $p(\epsilon')$ and ϵ' are taken from Fig. 3 (b). In that calculation it was also necessary to locally determine the tangent to the Poisson adiabat (in order to construct the wave adiabat (B–C)). The slope of the tangent is $\partial p/\partial X|_\sigma \equiv -n^2 c_s^2/(1 - c_s^2)$ and this relation can be easily inverted to yield the local velocity of sound $c_s(\epsilon')$ required for the integration in (32).

In the rest frame of incoming matter, the velocity of matter on the simple wave is $v'_w(\epsilon_w) = [v''_w(\epsilon_w) + v_{CJ}]/[1 + v''_w(\epsilon_w) v_{CJ}]$, and consequently in the CM frame $v_w(\epsilon_w) = [v'_w(\epsilon_w) - v_{CM}]/[1 - v'_w(\epsilon_w) v_{CM}]$. The complete simple wave profile is obtained by successively increasing ϵ_w in (32) until v_w becomes zero. Then, the simple wave has completely decelerated matter in the central region.

The similarity variable ζ corresponding to a given ϵ_w is determined via [1]

$$\zeta = \frac{v_w(\epsilon_w) + c_s(\epsilon_w)}{1 + v_w(\epsilon_w) \, c_s(\epsilon_w)} \; . \tag{33}$$

The final energy density ϵ where matter comes to rest determines the position $\zeta \equiv c_s(\epsilon)$ of the head of the simple wave. Profiles of T^{00}/ϵ_0 and T as functions of ζ are shown in Fig. 5 for $v_{CM} = 0.8$.

Point B on the generalized shock adiabat is numerically determined as $p_B \simeq 2.478\,\epsilon_0$, $X_B \simeq 0.198\,X_0$ ($\epsilon_B \simeq 18.271\,\epsilon_0$, $n_B \simeq 10.236\,n_0$). In the rest frame of matter emerging from the shock (O–A), the velocity at the head of the simple wave is $v''_w(\epsilon_B)$ as given by (32), and thus, in the rest frame of incoming matter, $v_B \equiv [v''_w(\epsilon_B) + v_{CJ}]/[1 + v''_w(\epsilon_B) v_{CJ}] \simeq 0.820$. Therefore, the CM velocity required to reach point B is $v_{CM} \equiv v_B$.

Above point B and below C, the compression profile consists of a shock to the CJ point A, a compressional simple wave, and a second shock. The position of the first shock and the respective hydrodynamic quantities are determined as above. In the rest frame of the second shock, the velocities obey the relations (25), except that quantities with subscript 0 have to be replaced by the quantities $\tilde{\epsilon}$, $\tilde{p}$, and $\tilde{v}$. In the rest frame of compressed matter, the second shock moves with velocity

$$v_{sh} \equiv -v = \left[\frac{(p - \tilde{p})(p + \tilde{\epsilon})}{(\epsilon - \tilde{\epsilon})(\tilde{p} + \epsilon)} \right]^{1/2} \; , \tag{34}$$

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while matter flows into that shock with velocity

$$v_w(\tilde{\epsilon}) \equiv \frac{\tilde{v} - v}{1 - \tilde{v}\,v} = -\left[\frac{(p - \tilde{p})(\epsilon - \tilde{\epsilon})}{(p + \tilde{\epsilon})(\tilde{p} + \epsilon)}\right]^{1/2} . \qquad (35)$$

Using (32) (after boosting it to the global CM frame) for the left hand side, this equation represents a condition to determine $\tilde{\epsilon}$, $\tilde{p}$. In practice, one constructs the compressional simple wave starting from the CJ point as in the previous case, but simultaneously checks for each $v_w(\epsilon_w)$ whether the condition (35) is fulfilled. At first, the left hand side will be *larger* than the right due to the following reason: a compressional wave which is too "short" implies a shock that is too strong (the corresponding final state lies above the true final state on the wave adiabat), and a stronger shock is more effective in decelerating matter. For a given v_{CM}, a shock that is too strong will result in a net *positive* velocity of matter in the final compressed state, instead of bringing it only to rest, or in other words, $v_w(\tilde{\epsilon})$ was not yet sufficiently *negative* in order that matter is just stopped by the second shock. Analytic profiles for the hydrodynamic variables are shown in Fig. 6 for $v_{CM} = 0.825$.

Point C is numerically determined as $p_C = 2.703\,\epsilon_0$, $X_C = 0.195\,X_0$ ($\epsilon_C = 18.944\,\epsilon_0$, $n_C = 10.525\,n_0$). From this point onwards, single shock solutions are again hydrodynamically stable. Thus, from (26) we determine the CM velocity to reach that point as $v_{CM} \equiv v_C \simeq 0.831$. Profiles for the hydrodynamic variables are presented in Fig. 7 for $v_{CM} = 0.9$.

This concludes the discussion of the analytic construction of the compression wave profiles. We finally note that in case point A of the Taub (or generalized shock) adiabat is not a true CJ point but a kink in the adiabat[3], also a double shock solution is possible for certain values of v_{CM} [4]. Since this situation does not occur for the EoS considered here, we do not discuss it in detail.

4   Numerical results

In this section we present numerical solutions for the compression of nuclear matter. The first test is whether the tables of thermodynamic quantities constructed as described in Section 2 are sufficiently accurate to reproduce the *analytical* solution presented in the preceding section. To this end, we solved the Taub equation (24) and constructed the generalized shock adiabat *using linear interpolation on these tables*. The curves of Fig. 3 are satisfactorily reproduced, even the wave adiabat, the construction of which involves the simultaneous solution of eqs. (24) and (31). The position of the CJ point can be found within 1% accuracy.

Then we checked whether the analytic shape of a simple compression wave can be reproduced. To this end, a discretized form of the thermodynamical identity (19) using tabulated values for the pressure was employed in the integrand of eq. (32). Also in this case, agreement was found to be rather good, confirming that the tabulation of the thermodynamic quantities is sufficiently accurate.

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[3]This could happen for particularly "stiff" hadron matter equations of state. Then, this point corresponds to the phase boundary between hadron and mixed phase matter.

The SHASTA and relativistic HLLE algorithms were described in detail in Ref. [1]. To test their numerical performance in the "slab-on-slab" collision problem, we choose the four different CM velocities $v_{CM} = 0.7$, 0.8, 0.825, and 0.9, for which the analytical profiles were explicitly constructed as discussed in Section 3. The numerical profiles of the CM frame energy density T^{00} (normalized to the ground state energy density ϵ_0) and the temperature T (in MeV) are shown in Figs. 4 (a–c) for the relativistic HLLE ($\Delta t/\Delta x \equiv \lambda = 0.99$) and in Figs. 4 (d–f) for the SHASTA ($\lambda = 0.4$) for $v_{CM} = 0.7$. (The presentation of quantities in terms of the similarity variable ζ is advantageous since in this way one can easily monitor the approach of the numerical to the analytical solution [1].) The resolution of the shock front is rather good (~ 4 grid cells) for both algorithms already after a few time steps (~ 50). The SHASTA produces a small overshoot at the shock front.

In Fig. 5 we show the corresponding results for $v_{CM} = 0.8$. The approach to the analytical solution is slow (about 500 time steps for both algorithms). At early times the compression configuration rather resembles a shock front that is broadened by viscosity [20]. The SHASTA run features a non-propagating instability on the shock plateau at $x \simeq 25$ (as a function of $\zeta = x/t$, such an instability moves of course to the left in the course of time, cf. Fig. 5 (e)). This instability can be removed by decreasing the antidiffusion fluxes, see Appendix. The HLLE run shows a slowly decaying overshoot in the energy density around $x = \zeta = 0$. It is a remnant of the first few time steps when numerical transients cause an overestimate of the shock plateau (this effect is clearly visible in Figs. 4 and 7). Apart from these phenomena, both algorithms reproduce the position and shape of the compressional wave configuration quite well after about 500 to 1000 time steps.

The origin of the instability (Fig. 5 (d)) and the overshoot (Fig. 5 (a)) and the reason for their persistence are easily understood considering the fact that the final state corresponds to TA mixed phase matter where pressure gradients are small (cf. Fig. 2 (a); the same holds for temperature gradients, cf. Fig. 2 (b), therefore, corresponding phenomena do not occur in the temperature profiles, Figs. 5 (c,f)). On one hand, this facilitates compression of matter (one has to exert less force in the compression) and subsequently the creation of local density maxima. On the other hand, the usual driving force for the expansion of local density maxima is absent. They can only decay on account of numerical diffusion. Therefore, the instability produced by the standard version of the SHASTA (Figs. 5 (d,e)) is removed when the antidiffusion is decreased, since this increases the numerical diffusion. On the other hand, the numerical diffusion of the relativistic HLLE is just large enough to slowly damp out the overshoot at $x = \zeta = 0$ (Figs. 5 (a,b)). We finally note that in the mixed phase a density increase (along the simple compression wave) corresponds to a temperature decrease, cf. Fig. 5 (b,c,e,f), reflecting once more the TA nature of this phase.

Fig. 6 shows our results for $v_{CM} = 0.825$. While the relativistic HLLE is well able to reproduce the rather complex compressional wave configuration (modulo unavoidable numerical dissipation), the SHASTA fails completely: apart from a slowly growing, non-propagating instability at $x \simeq 10$, a single shock front forms instead of the configuration consisting of two shocks and a simple wave. The reason is again that the numerical dissipation is too small (see also Appendix). Reducing the *anti*diffusion in the SHASTA therefore considerably improves the reproduction of the analytical profile, see Appendix for details.

Finally, in Fig. 7 the profiles are shown for $v_{CM} = 0.9$. Since the hydrodynamically stable solution is again a single compression shock, this case is similar to Fig. 4, i.e., both algorithms reproduce the analytical solution rather well after a few time steps. Comparing Figs. 7 (b) and (e), the resolution of the shock front in the SHASTA appears to be worse than for the HLLE. However, one has to remember that, due to the smaller CFL–number λ for the SHASTA run, the profiles extend over fewer cells after the same number of numerical time steps [1]. Close inspection reveals that the shock front is resolved over ~ 3 grid cells for *both* algorithms after about 30 time steps.

In conclusion, despite the simplicity of the numerical algorithms the (partly rather complex) analytical solutions are reproduced remarkably well with the relativistic HLLE and also with the SHASTA after decreasing the antidiffusion. This constitutes an independent "numerical" proof of the correctness of the arguments presented in [4] and Section 3. It is remarkable that an usually unwanted feature like numerical diffusion is now *necessary* to reproduce the physical solution. The reason is that in TA matter this solution reacts sensitively to numerical instabilities. In order to suppress the latter and consequently make the transport algorithm more robust, one has to increase the numerical diffusion. In the case of single compression shocks the approach to the analytical solution requires about 50 numerical time steps. However, to reproduce the more complex compressional wave configurations for TA matter takes about a factor 10 to 20 longer. Consequences for the simulation of heavy–ion collisions are pointed out in the next section, where the compression of finite systems is studied. We note that for the HLLE runs we have used a constant velocity of sound $c_s^2 = 1/3$ in the signal velocity estimates, cf. also [1]. Using the physical sound velocity has already been shown to produce unphysical solutions in the expansion of matter into vacuum [1]. As demonstrated in the Appendix, such problems occur also in the compression [4].

5 Compression of finite systems

We consider a one–dimensional collision of two (equal) nuclei with rest frame radius R. The nuclei have the CM velocity v_{CM} and are initialized in the moment of first contact. The initial condition for this problem is

$$\epsilon(x,0) = \begin{cases} \epsilon_0 , & |x| \leq 2\,R/\gamma_{CM} \\ 0 , & |x| > 2\,R/\gamma_{CM} , \end{cases} \tag{36}$$

$$n(x,0) = \begin{cases} n_0 , & |x| \leq 2\,R/\gamma_{CM} \\ 0 , & |x| > 2\,R/\gamma_{CM} , \end{cases} \tag{37}$$

$$v(x,0) = \begin{cases} v_{CM} , & -2\,R/\gamma_{CM} \leq x \leq 0 \\ -v_{CM} , & 0 < x \leq 2\,R/\gamma_{CM} \\ 0 , & |x| > 2\,R/\gamma_{CM} . \end{cases} \tag{38}$$

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[4] Of course, the choice $c_s^2 = 1/3$ is only safe as along as the *physical* velocity of sound does not *exceed* this value. For instance, for *pure* hadronic matter described by the $\sigma - \omega$–model the velocity of sound even approaches the causal limit for large ϵ, n. Fortunately, for the EoS constructed in Section 2, the phase transition to the QGP phase (with $c_s^2 = 1/3$) prevents this, and no problems were encountered for the test cases studied here.

The compression ends when the compression front has traversed the nucleus. In the CM frame, this corresponds to a time [4]

$$t_F = \frac{\gamma_{CM}(1 - v'_{sh}v_{CM})}{v'_{sh}} \, R \quad , \tag{39}$$

where v'_{sh} is the velocity of the compression front in the rest frame of the incoming nucleus as calculated in Section 3, i.e., it is either the velocity of the single compression shock in the case $v_{CM} \leq v_{CJ}$ or $v_{CM} \geq v_C$, or that of the shock to the CJ point for $v_{CJ} < v_{CM} < v_C$. In the latter case, $v'_{sh} \simeq 0.878$, see Section 3. In Fig. 8 t_F (in units of R) is shown as a function of v_{CM}. If v_{CM} is less than the velocity of sound in the ground state of nuclear matter $c_{s,0} \equiv (K_0/9\mu_0)^{1/2}$ ($\simeq 0.190$ for the parameters of Section 2), no shock discontinuity can form [19]. The point where the generalized shock adiabat enters the mixed phase and also point C on this adiabat (cf. Fig. 3 (b)) are visible as kinks in t_F.

In order for the numerical algorithms to approach the correct analytical solution for times $t < t_F$, i.e., before expansion sets in, the grid spacing has to be sufficiently small. We have seen in the previous section that in the case of single compression shocks the shock profile is accurately reproduced after about 50 time steps. Therefore, the grid spacing should be chosen smaller than $\Delta x \simeq t_F/50\,\lambda$. To give an example, for $t_F \simeq R$, $\Delta x < 0.02\,R$ for the HLLE ($\lambda = 0.99$) and $\Delta x < 0.05\,R$ for the SHASTA ($\lambda = 0.4$). For a typical nuclear radius of $R \simeq 5$ fm, a feasible grid spacing for the HLLE would therefore be $\Delta x < 0.1$ fm and for the SHASTA $\Delta x < 0.25$ fm. However, for ultrarelativistic collisions where $t_F \sim 0.2\,R$, Δx should at least be a factor of 5 smaller.

We do not show explicit calculations for the case of compression via single shock discontinuities, since the result is obvious: after the incident nuclei are completely stopped by the compressional shock waves, expansion sets in. The only remaining question is whether the performance of the algorithms for this *expansion* of *baryon-rich* nuclear matter with a *realistic* nuclear matter EoS is of similar quality as for the (net) baryon-free case studied in Ref. [1]. This investigation is out of the scope of the present work.

For the complex compressional wave patterns of Figs. 5, 6 we have to good approximation $t_F \sim 0.57\,R$ for all cases, cf. Fig. 8. We have seen that of the order of 500 time steps are necessary to approach the analytical profiles. Thus, the grid spacing should be chosen smaller than $0.001\,R \simeq 0.005$ fm for the HLLE and smaller than $0.003\,R \simeq 0.015$ fm for the SHASTA. The numerical effort to calculate a collision on such a fine grid is definitely prohibitive, especially for multi–dimensional problems. It is therefore of interest whether the distortion caused by a too coarse grid spacing really changes physical observables, such as particle spectra, in the final state.

The calculation of these spectra requires assumptions about the particle emission process from the fluid and, ultimately, about the "freeze–out" of the fluid. This is out of the scope of the present work and will be subject of a forthcoming paper [21]. For the moment, we only study the spatial CM energy density distribution of the *fluid*, which serves as input for a "freeze–out" calculation. We compare the time evolution of this quantity for one–dimensional collisions of finite nuclei at $v_{CM} = 0.8$, calculated with the HLLE for $\Delta x = 0.01\,R$ and $\Delta x = 0.001\,R$ (Fig. 9), and calculated with the SHASTA for $\Delta x = 0.025\,R$

and $\Delta x = 0.0025\,R$ (Fig. 10). For the SHASTA runs we have reduced the antidiffusion fluxes, since this stabilizes the algorithm and improves results considerably, cf. Appendix. Let us note that energy, momentum, and baryon number conservation for all runs is better than 10^{-3}.

In Fig. 9 one observes that for the compressional stage, Figs. 9 (a,b), the finer grid spacing leads, as expected, to a much more accurate description of the analytical profile, cf. Fig. 5. However, the final CM energy density profiles emerging after the expansion stage, Figs. 9 (c,d), are quite similar. Indeed, the run with the coarser grid spacing even yields a smoother profile and does not show "terraces" as can be seen in (b,d) on the low density tails in either compression or expansion stage. Furthermore, small–scale numerical instabilities are observed around the stationary point of the profile due to the exceedingly long calculation time in (d). Consequently, for the HLLE the use of a grid spacing which is sufficiently fine to reproduce the compressional stage yields no obvious advantage when one is interested in final state fluid quantities, but it is even disadvantageous from the point of view of the required calculational effort (the same physical time corresponds to 10 times the number of numerical time steps, the smaller grid spacing requires 10 times more grid cells on the same physical length scale).

From the physical point of view we note that the expansion of the system proceeds similar as in the (net) baryon-free case studied in [1]: the final compressed state consists of TA mixed phase matter, and is therefore consumed by a rarefaction discontinuity. In Figs. 9 (c,d) one clearly observes this shock wave travelling into the high density zone. Hadronic matter is expelled from the shock and subsequently expands via a simple rarefaction wave.

For the SHASTA (Fig. 10) the distorting effect of a too coarse grid spacing is more obvious. Clearly, not only is the reproduction of the compression profile insufficient, also the rarefaction shock wave in the expansion stage is no longer discernible. On the other hand, the run with the finer grid spacing $\Delta x = 0.0025\,R$ produces excellent results, although there exists again a tendency to produce "terraces" on the tail of the hadronic expansion wave, as for the HLLE run with $\Delta x = 0.001\,R$.

However, as far as the final state profiles and particle spectra at freeze–out are concerned, all runs would give similar results. It is therefore questionable whether high precision calculations with a small grid spacing are really compulsory for the simulation of heavy–ion collisions. We note that results for the case $v_{CM} = 0.825$ are rather similar, wherefore we do not show them explicitly.

6 Conclusions

In this paper we have studied the compression of nuclear matter in one–dimensional hydrodynamical "slab-on-slab" collisions. First, a nuclear matter EoS was constructed, which describes the hadron matter phase in terms of an improved version of the original $\sigma - \omega$– model. For the QGP phase, the MIT bag model EoS was employed, and both phases were matched via Gibbs' phase equilibrium conditions for a first order phase transition. In numerical algorithms to solve relativistic hydrodynamics, this EoS must be used in tabulated

form, since an "on-line" determination of the pressure (which is required in the solution of the hydrodynamical equations at various intermediate steps, cf. [1]) would be calculationally prohibitive. However, the usefulness of the tables for thermodynamic quantities is not restricted to the hydrodynamical problems studied here, they can also be applied to future and, most important, more realistic multi–dimensional calculations.

Subsequently we have constructed the analytical solution to the one–dimensional compression of nuclear matter for various incident velocities. Since nuclear matter with a first order phase transition is TA in certain regions of the independent thermodynamical variables, the hydrodynamically stable compression solution is no longer a single shock discontinuity, as for TN matter, but consists in general of a sequence of shocks and compressional simple waves. We remark that the constancy of the specific entropy on compressional simple waves leads to a plateau in the respective excitation function which might in turn lead to a corresponding plateau in the excitation function of the pion multiplicity [22]. This would be a clear signal for the phase transition between hadron matter and the QGP.

Then we have investigated the performance of numerical algorithms for this particular test problem. Both the relativistic HLLE and the SHASTA algorithm are well able to reproduce the analytical profiles, provided the numerical diffusion is sufficiently large. In a sense, this constitutes an independent numerical "proof" for the correctness of the theoretical arguments presented in Ref. [4] and in Section 3 of this work concerning the hydrodynamically stable compression solution. The algorithms require about an order of magnitude longer to approach the more complicated compressional solution in the region where nuclear matter is TA than they need for the reproduction of the compressional shock waves in TN matter. To account for this, one has to choose a sufficiently fine grid spacing Δx in collisions of finite nuclei at the respective incident energies, in order that the analytical profile is reproduced before the compression wave has traversed the nucleus and expansion sets in. However, we have found that the final profiles for a calculation employing a too coarse Δx do not differ significantly from those for a finer Δx, so that the computational effort can be considerably reduced, at least as long as one is interested in final state observables only[5].

We anticipate that the algorithms presented in detail in Ref. [1] and studied for various non–trivial test problems in [1] and the present work will find numerous applications in multi–dimensional hydrodynamical problems. Of special interest is a deeper understanding of the flow as discovered in BEVALAC experiments a decade ago [24] and just recently confirmed by quantitatively excellent data from the EOS–collaboration [25]. Flow was also observed in recent AGS experiments [26]. At present it is unclear whether its magnitude can be completely accounted for in the framework of microscopic cascade models [27] or whether it is consistent only with a phase transition in the nuclear matter EoS [28].

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[5]However, the effect will be negligible even on observables that are most sensitive to the *initial* hot stage, like photons or dileptons [23], since the *average* temperatures are also very similar for the various runs of Figs. 9, 10.

A   Appendix

In this Appendix we study the effect of modifications of the algorithms as used in the main part of this work. First, we demonstrate that the performance of the SHASTA in the reproduction of the complex compressional wave configurations of Figs. 5, 6 is considerably improved by decreasing the antidiffusion fluxes. As discussed in the Appendix of [1], to this end the first factor 1/8 in eq. (17) of [1] is replaced by 1/10. The effect is shown in Fig. 11. One observes that the instabilities in Figs. 5, 6 (a,b,d,e) are removed due to the larger numerical dissipation introduced by reducing the antidiffusion fluxes. Contrasting Fig. 6, the wave configuration for $v_{CM} = 0.825$ is now in reasonable agreement with the analytical profile.

Finally, we discuss the effect of using the physical velocity of sound in the signal velocity estimates for the relativistic HLLE. In Fig. 12 we confront the correct solution for (a) $v_{CM} = 0.8$ and (b) $v_{CM} = 0.825$ obtained with a constant velocity of sound $c_s^2 = 1/3$ in the signal velocity estimates (cf. eqs. (29, 30) of Ref. [1]) with the corresponding results (c,d) using the physical sound velocity calculated according to (19) on the table of pressure values. Comparing (a) and (c) one observes that instead of reproducing the simple compression wave the code now grossly overestimates the strength of the first shock which even accelerates matter to positive velocities. Subsequently, a second *rarefaction* shock brings matter to rest. The final state has a smaller energy density than the correct solution.

A comparison of (b) and (d) reveals that the code fails completely to reproduce the configuration consisting of a compressional simple wave between two shocks. Instead, a single compressional shock is formed which does not decay, as it should for TA matter. The situation is rather similar to what the standard version of the SHASTA yields for this case (Fig. 6 (e)). A too small diffusion was then identified as the reason for the failure to produce the analytical result. This is a natural explanation also for understanding Fig. 11 (d): the physical sound velocity in the mixed phase is small, and a small sound velocity decreases the signal velocity estimates [1], and consequently, the numerical diffusion.

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Figure Captions:

Fig. 1: Nuclear matter phase diagram in the (a) $T - \mu$, (b) $T - n/n_0$, (c) $\epsilon/\epsilon_0 - T$, and (d) $\epsilon/\epsilon_0 - n/n_0$ plane. The dotted line in (c) corresponds to the $n = 0$–axis in (b), the dotted line in (d) to the $T = 0$–axis in (c).

Fig. 2: (a) p/ϵ_0, (b) T, (c) μ, and (d) s/n_0 as functions of ϵ/ϵ_0 and n/n_0. (For presentation purposes, only every second meshpoint on the 201×240 mesh is plotted.)

Fig. 3: (a) Single shock Taub adiabat with the center (X_0, p_0) (point O) and the CJ point A. (b) Generalized shock adiabat. The part (O–A) and that above C are identical to the Taub adiabat in (a). Part (A–B) is a Poisson adiabat (B is the inflection point of this adiabat), part (B–C) the wave adiabat (for details concerning its construction see text). The dotted line is the unstable part of the Taub adiabat of Fig. 3 (a).

Fig. 4: "Slab-on-slab" collision for $v_{CM} = 0.7$ as calculated with (a–c) the relativistic HLLE algorithm ($\Delta x = 1$, $\lambda = 0.99$) and (d–f) the SHASTA ($\Delta x = 1$, $\lambda = 0.4$). (a,d) CM frame energy density T^{00} (normalized to the ground state energy density ϵ_0) as a function of x for $10, 20, 30, ..., 100$ time steps, (b,e) T^{00}/ϵ_0 and (d,f) temperature T as functions of the similarity variable $\zeta \equiv x/t$, for 10 (stars), 20 (squares), 30 (diamonds), 50 (dotted line), and 100 time steps (dashed line) in comparison to the analytical result (full line).

Fig. 5: The same as in Fig. 4 for $v_{CM} = 0.8$. In (b,c,e,f) stars correspond to 50, squares to 100, diamonds to 200, the dotted line to 500, and the dashed line to 1000 time steps.

Fig. 6: The same as in Fig. 5 for $v_{CM} = 0.825$.

Fig. 7: The same as in Fig. 4 for $v_{CM} = 0.9$.

Fig. 8: The CM time t_F the compression front needs to reach the edge of the incoming nucleus as a function of v_{CM}.

Fig. 9: The CM frame energy density (in units of ϵ_0) as a function of x (in units of the nuclear radius R in its rest frame) for a one–dimensional collision of finite nuclei at $v_{CM} = 0.8$, calculated with the relativistic HLLE ($\lambda = 0.99$) for (a,c) $\Delta x = 0.01\,R$ and (b,d) $\Delta x = 0.001\,R$. (a,b) show profiles at constant CM time in the compression stage (for (a) the time steps are $0, 20, 40, ..., 100$, for (b) $0, 200, 400, ..., 1000$), (c,d) the subsequent expansion (for (c) the time steps are $120, 140, ..., 240$, for (d) $1200, 1400, ..., 2400$). For the sake of clarity, profiles are alternatingly shown as full and dotted lines.

Fig. 10: The same as in Fig. 9 for the SHASTA ($\lambda = 0.4$) with (a,c) $\Delta x = 0.025\,R$ and (b,d) $\Delta x = 0.0025\,R$.

Fig. 11: The effect of reducing the antidiffusion fluxes in the SHASTA, (a–c) as in Figs. 5 (d–f), (d–f) as in Figs. 6 (d–f).

Fig. 12: The effect of using the physical velocity of sound in the signal velocity estimates in the relativistic HLLE (c,d) as compared to the correct solution (a,b) (Figs. (a,b) are identical to Figs. 5, 6 (b)).

RHLLE

(a)

(b)

(c)

SHASTA

(d)

$v_{CM}=0.7$

(e)

(f)

anal.

it = 10

it = 20

it = 30

it = 50

it = 100

Figure panels:

- (a) RHLLE, T^{00}/ε_0 vs X
- (b) T^{00}/ε_0 vs ζ
- (c) T [MeV] vs ζ
- (d) SHASTA, T^{00}/ε_0 vs X, $v_{CM}=0.8$
- (e) T^{00}/ε_0 vs ζ
- (f) T [MeV] vs ζ

Legend:
anal.
it = 50
it = 100
it = 200
it = 500
it = 1000